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COMMENT

A comment on 'An important equation for the Anderson model'

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Abstract. The recent claim of a new, generally exact relationship for the Anderson model is shown to be incorrect. In particular, the relationship is shown not to be correct for all band fillings for the $U = 0$ Anderson model.

The Anderson model is given by

$$H = \sum_{\boldsymbol{k},\sigma}(\epsilon_k - \epsilon_f)a^+_{\boldsymbol{k},\sigma}a_{\boldsymbol{k},\sigma} + \sum_{\sigma}(\epsilon_d - \epsilon_f)c^+_{d\sigma}c_{d\sigma} + Un_{d_\downarrow}n_{d_\uparrow} + \sum_{\boldsymbol{k},\sigma} V_{\boldsymbol{k}d}\left(a^+_{\boldsymbol{k},\sigma}c_{d\sigma} + c^+_{d\sigma}a_{\boldsymbol{k},\sigma}\right) \tag{1}$$

where we have adopted the notation of Teng (1995). Defining the Zubarev double-time Green functions (DTGFs)

$$\Gamma^{(1)}_{dd,\sigma}(\omega + \epsilon_f) \equiv \langle\langle \mu_\sigma ; c^+_{d\sigma}\rangle\rangle(\omega + \epsilon_f) \tag{2}$$

$$F_{dd,\sigma}(\omega + \epsilon_f) \equiv \langle\langle \mu_\sigma ; \mu^+_\sigma\rangle\rangle(\omega + \epsilon_f) \tag{3}$$

where $\mu_\sigma \equiv n_{d_{-\sigma}}c_{d_\sigma}$, Teng claimed that the relationship

$$F_{dd,\sigma}(\omega) = \Gamma^{(1)}_{dd,\sigma}(\omega) \tag{4}$$

is exactly true for all values of $\langle n_{d\sigma}\rangle$ and all values of the parameters of the Anderson model. The purpose of this comment is to show that the claim that (4) is generally true for all U and all band fillings is incorrect. In particular we show that in the $U = 0$ case, where

$$\langle \mu^+_{-\sigma}a_{\boldsymbol{k}-\sigma}\rangle = \langle a^+_{\boldsymbol{k}-\sigma}\mu_{-\sigma}\rangle = \langle n_{d\sigma}\rangle\langle c^+_{d-\sigma}a_{\boldsymbol{k}-\sigma}\rangle \tag{5}$$

holds exactly, the presumption of (4) requires $\langle n_{d-\sigma}\rangle = \frac{1}{2}$ and therefore (4) is not generally correct for all U and all band fillings.

One can readily form two important identities for the Anderson model,

$$\langle\{[\mu_\sigma, H], c^+_{d_\sigma}\}\rangle = (\epsilon_d - \epsilon_f + U)\langle n_{d-\sigma}\rangle + \sum_{\boldsymbol{k}} V_{\boldsymbol{k}d}(\langle c^+_{d_{-\sigma}}a_{\boldsymbol{k},-\sigma}\rangle - \langle a^+_{\boldsymbol{k},-\sigma}c_{d_{-\sigma}}\rangle) \tag{6}$$

$$\langle[\mu_\sigma, H], \mu^+_\sigma\rangle = (\epsilon_d - \epsilon_f + U)\langle n_{d-\sigma}\rangle + \sum_{\boldsymbol{k}} V_{\boldsymbol{k}d}(\langle \mu^+_{-\sigma}a_{\boldsymbol{k},-\sigma}\rangle + \langle a^+_{\boldsymbol{k},-\sigma}\mu_{-\sigma}\rangle - \langle a^+_{\boldsymbol{k},-\sigma}c_{d-\sigma}\rangle). \tag{7}$$

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Using (4) to obtain the LHS of (6) and (7) gives

$$\langle\{[\mu_\sigma, H], c_{d_\sigma}^+\}\rangle = \langle\{[\mu_\sigma, H], \mu_\sigma^+\}\rangle \tag{8}$$

i.e. if (4) is true we must have

$$\sum_{\boldsymbol{k}} V_{\boldsymbol{k}d}\langle c_{d_{-\sigma}}^+ a_{\boldsymbol{k},-\sigma}\rangle = \sum_{\boldsymbol{k}} V_{kd}(\langle \mu_{-\sigma}^+ a_{\boldsymbol{k},-\sigma}\rangle + \langle a_{\boldsymbol{k},-\sigma}^+ \mu_{-\sigma}\rangle). \tag{9}$$

In the $U = 0$ case, (5) gives, when used in (9),

$$(1 - 2\langle n_{d_{-\sigma}}\rangle)\sum_{\boldsymbol{k}} V_{\boldsymbol{k}d}\langle c_{d-\sigma}^+ a_{\boldsymbol{k},-\sigma}\rangle = 0. \tag{10}$$

Thus, in the $U = 0$ case, the presumed validity of (4) requires

$$\langle n_{d_\uparrow}\rangle = \langle n_{d_\downarrow}\rangle = \tfrac{1}{2} \tag{11}$$

and (4) is, therefore, not generally true for all band fillings and all values of the Anderson Hamiltonian parameters.

References

Teng B 1995 *J. Phys.: Condens. Matter* **7** 867